Study on the 3D Digital Model of Chinese Fir Based on the Crown Profile Curve

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Abstract: In the process of forest management, the crown profile model can not only directly reflect crown size and crown volume but also can be used to derive forest stand visualization. We used a total of 410 trees collected from 98 pure even-aged temporary sample plots established in Fujian Province to develop crown profile models of Chinese fir (Cunninghamia lanceolata). To describe the crown more accurately, three different curve equations were selected to describe the crown profiles of different age groups (i.e., young, middle and mature forests). In the model fitting process, a continuous autoregressive error structure (CAR) was used to account for the correlation between repeated measurement data from the same tree crown and a weighted regression function was used to eliminate the heteroscedasticity. Based on the developed crown profile models, we proposed a new approach which used the theory of Directed Tree to describe the semantic relation of morphological structure and constructed a digital model to express each tree in forest stand. The results showed that the predicted accuracy of model fit by the CAR had better fitting effects and the residual distribution of which showed the characteristics of equal variance and unbiasedness. The 3D digital tree constructed in this paper was more consistent with the actual growth of plantation and was of great significance for qualitative and quantitative analysis of tree competition level.

1. Introduction

Tree crowns play an essential role in plantation. Crown structure is the result of the interaction between tree and the environment. The size (crown width, crown length, crown outer shape and crown volume) can not only directly reflect tree vigour and the degree of canopy overlap among individuals in forests, but also can be used to assess competitive level.

A variety of equations with tree factors (i.e., diameter at breast height, height and crown variables) have been used to describe crown profiles. Many scholars[2] suggested the crown shapes to be simple regular geometrical curves (i.e. cylinder, parabola, cone, neiloid). However, crown shapes of different tree species and which in different growth periods are not rigid. Many flexible equations were used to express the crown shapes, such as polynomial model, power function, S-curve, modified Beta curve equation, segmented equation and exponential equation. The coefficients of these crown profile models were estimated by ordinary least square (OLS) or nonlinear least square (NLS). However, crown measurements from the same tree are related. This destroys the independence between observations and invalidates the basic hypothesis of the least square framework[3]. A mixed effect method with random terms was used to consider autocorrelation and heteroscedasticity between model error terms. But secondary sampling is required to calculate the random parameters, which limits the application of the mixed effects model method.

In recent years, visualization technology has been used to show the tree crowns of plantation. In earlier studies, several scholars described tree morphogenesis and growth process based on tree growth knowledge and mechanism, such as L-system, fractals and Particle system. Tree photos were also used to construct forest stand scenes. However, when viewed from multiple angles, it had the insignificance of "all trees exactly the same." Furthermore, some scholars used simple geometrical combinations to express crown morphology at different growth stages. However, the crown shapes of different tree species are not fixed, and the same tree species are different at different times. To reduce the burden of computer graphics rendering, Dong[1] simulated the crown shape of Chinese fir by rotating the crown shape curve 360 degrees around the stem axis. This method can accurately describe crown shapes in different growth conditions, but it is difficult to distinguish the tree species. Researchers constructed a set of interactive methods based on the real and interactive 3D virtual trees to simulate the forest management effect under different tending measures. Therefore, how to solve the contradiction between the authenticity of trees and large-scale management is still a challenge.

Chinese fir (Cunninghamia lanceolata) is an important timber tree species in the southern of China. The dynamic knowledge of Chinese fir is significant for forest protection and management in the region. With few exceptions, the crown profile models of Chinese fir have all been estimated with the NLS or the OLS. The objective of this study was to develop a crown profile model for even-aged Chinese fir in China.

2. Study Area and Data

The study area was located in the Dali and Lanxia forest farms in Fujian province, China. Data were collected from 98 pure even-aged temporary sample plots (20 m×30 m) established in pure Chinese fir plantations according to different age groups and stand densities. 4~5 living and undamaged trees were selected from each plot and a total of 410 trees were collected. A crown measuring device which consists of a transparent drawing board, a tripod and a diopter (Fig. 1) was used to measure each selected trees. The device is used as follows:

(1) According to the principle of similar triangle, at a certain distance from the measured tree, the drawing board is fixed on the tripod to keep the board perpendicular to the ground. Then the tripod is moved backwards until the whole tree crown can be observed through the board.

(2) In order to draw the tree crown profile on the drawing board with a pen. Firstly, point A at the top of the tree crown and point B at the base of the tree crown are drawn. Then, the sketch of the tree crown is drawn from point A to point B. Line segment EF from Point E to point F represents the maximum crown width is identified.

(3) The crown length LCL (LCL=A'B', m) and crown width (CW=E'F', m) of the tree crown were measured by height measuring instrument and tape. The crown height (CHi, m) at the position of i*LCL (i=1/10, 1/4, 1/2, 3/4, 9/10) and the crown radius (CRi, m) corresponding to CHi. were calculated respectively according to the ratio of A'B' to AB and the ratio E'F' to EF. Each crown should be observed from several directions and take the mean measurement.

(4) The following measurements were also obtained for each selected tree: tree age (t, year); diameter at breast height (DBH, cm); total tree height (H, m); height above ground to the crown base (HCB, m). DBH was measured to the nearest 0.1cm and other tree variables were measured to the nearest 0.1m. Statistical data of Chinese fir crown survey collected were described in Table 1.

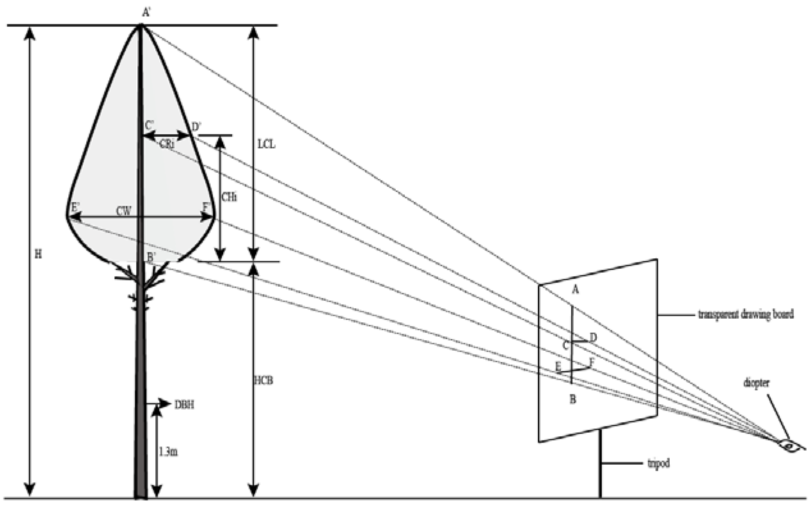

Fig.1 The Tree Crown Measurement Diagram of Chinese Fir.

Table 1 Summary Statistics Of Measurements of Tree Variables.

Tree variables	Mean	Min	Max	STD
t (years)	16	5	29	7.25
N (trees/hm^2)	1850	900	4000	60.39
DBH (cm)	16.4	5.9	33.2	5.52
H (m)	12.6	3.0	25.5	3.91
CW (m)	3.6	1.6	7.6	1.05
LCR (m)	1.8	0.8	3.8	0.53
LCL (m)	6.0	1.1	14.3	2.65
HCB (m)	6.4	0.3	16.3	3.17
CR (m)	1.1	0.1	3.8	0.56

Note: Min = minimum, Max= maximum, Mean = average value, and STD=standard deviation.

3. Method

3.1 Crown Profile Modeling

Crown shapes of the same tree species at different growth stages were not fixed. In the early growth stage, the stem grows vigorously but the lateral branches are not developed. Eq. 1 can be used to describe the crown shape of young Chinese fir in this stage. In the middle-aged stage, crown shapes can be expressed by the modified polynomial Eq. 2. When the forest stand enters the mature stage, tree crowns begin to close and crown shapes trend to flat top gradually. Crown shapes can be described by the exponential Eq. 3 which contains the variable of stand density.

The specific equations are as follows:

$$CR = LCR * (1 - RCH)^{a_0} \quad (1)$$

$$CR = LCR * (b_0 * \ log(N) * \left(\frac{RCH-1}{RCH+1}\right) + b_1(RCH - 1) \quad (2)$$

$$CR = LCR * (exp(c_0 + c_1 * RCH + (c_2 * \ log(N)) * RCH^2)) \quad (3)$$

Where, CR is the crown radius at different crown positions; LCR is the largest crown radius; RCH is relative crown radius value; N is the stand density; log is the logarithm to the base 10; a_0, b_0, b_1, c_0, c_1 and c_2 are parameters.

In the modeling process, we used a CAR method to solve the autocorrelation problem. The method corrects the correlation between the same crown data by adding a continuous autoregressive error structure to the fitting process of the least square framework. The error structure takes into account the distance between the measurements, which allows the model to be applied to irregularly spaced data. The specific form is as follows:

$$e_{ij} = \sum I_m \, p_k^{h_{ij}-h_{ij-k}} e_{ij-k} + \varepsilon_{ij} \quad (4)$$

Where, e_{ij} is the ordinary regression residual corresponding to the jth crown measurement point of the ith tree, and e_{ij-k} is the ordinary regression residual corresponding to the j-k crown measurement points of the ith tree. When j⩽k, Im is equal to 0 and when j>k, Im is equal to 1. p_k is a k-order continuous autoregressive parameter and $h_{ij} - h_{ij-k}$ is the distance between the jth measured value and the j-kth measured value in the same tree. ε_{ij} is an independent random error with a mean of zero.

To account for the heteroscedasticity in forestry modeling, we used a weighted function with power form to calculate the variances for the error structure (Eq. 4). For the crown profile models, $\delta^2 = \gamma LCR^q$ was used to model the error variances. By taking the natural logarithm, the linearized function was as follows: $ln\delta^2 = ln\gamma + qlnLCR$. Where, the parameter q was estimated using the OLS. Then the weighting factor was as follows: $weight = 1/\sqrt{LCR^q}$.

3.2 Model Comparison and Selection

We used all crown data to estimate the parameters and used a cross-validation method to evaluate the prediction accuracy of the crown profile models. The model effect was determined by the following indexes: the coefficients of determination (R2), the mean square (MSE), the root mean square (RMSE), the mean absolute error (MAE), and the mean deviation (MD). The specific forms were shown as Eq. 5 to Eq. 9. The optimal model was selected according to the principle of R2 maximum and RMSE minimum.

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y}_i)^2} \quad (5)$$

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \quad (6)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-1}} \quad (7)$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i| \quad (8)$$

$$MD = \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)}{n} \quad (9)$$

Where, y_i is the actual value of the ith dependent variable, $\hat{y}_i$ is the predicted value of the ith dependent variable, $\bar{y}$ is the average of the actual value of the dependent variable, and n is the number of samples.

3.3 Tree Growth Rules and Constraints

The tree body structure is divided into the ground and underground root parts. The size of the crown expansion space can be controlled by the outer crown profile curve. Tree growth rules and constraints are as follows.

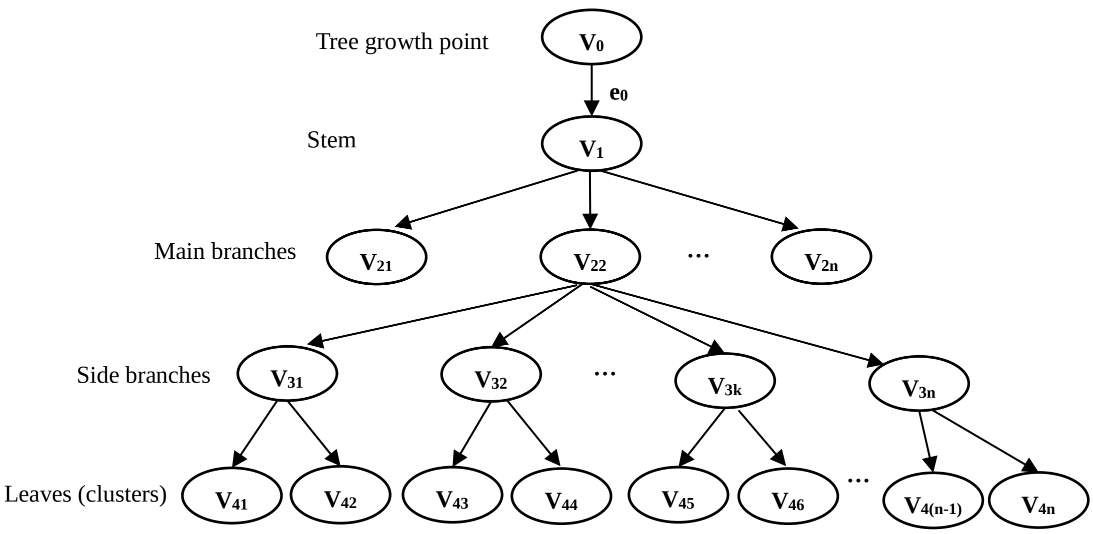

Fig.2 The Tree Topology Based on the Directed Tree.

3.3.1 Definition: Tree Entity

The tree is composed of stem, main branches, side branches, leaves and fruits. The tree is defined as a parent class and tree organs are considered to be subclasses that inherit from the parent class. Each subclass has its own geometric properties such as size, shape, distribution and texture.

Tree entity: {[Stem], [Crown], [Branch type, Branches, Leaves (clusters)], [Rules and Constraints]}

Stem: {[Tree position], [Tree height, Stem radius], [Texture images]}

Crown: {[Branches and leaves], [Crown length, Crown radius]}

Main branch: {[Side branches], [Position, Length, Radius, Azimuth], [Texture images]}

Side branch: {[Main branch], [Position, Length, Radius, Azimuth], [Texture images]}

Leaves (*clusters*): {[Blade shape, Position, Leaf inclination], [Texture images]}

3.3.2 Definition: Topological Relationship

The topological relationship between tree organs is as follows: Main branches are born on the stem, and are father nodes of the side branches. Leaves (clusters) are the child nodes of branches. These relationships can be described by a directed tree (G= (V, E)) in graph theory. Where, G is a binary group, V is a vertex set, E is a directed edge set. The topological relationship definition is shown in Fig. 2. V_0 is a root node of the directed tree. V_1 represents the stem node and e_0 is the connection relationship between V_0 and V_1. V_{2i} (i=1 ,…, n) represent main branches, and are the child nodes of V_1. V_{3j} (j=1 ,…, n) represent side branches, and are the child nodes of V_{2i}. V_{4k} (k=1,…, n) represent leaves (clusters), and are the end nodes of G.

3.4 Generation of 3d Digital Tree Based on Crown Profile Model

In this paper, we used the crown profile model to constrain the growth space of branches. The process of 3D digital tree generation was as follows.

3.4.1 Stem Generation Module

The radius of stem gradually increases from the tree top to the tree stump. The stem can be divided into n cylindrical geometries (0 Cylinder,1 Cylinder,…,n-1 Cylinder). Referring to the Domain Connected Graph (DCG) theory, the stem semantic model is defined as Eq. 10.

$$D_1 = \bigcup_{i=0}^{n-1} Cylinder_i(p, h, r_{i1}, r_{i2}) \quad (10)$$

Where, D_1 is the tree stem layer semantic model; $Cylinder_i$ is the ith cylinder; r_{i1} and r_{i2} are the top and base radius of the ith cylinder, respectively; p (x, y, z) is the coordinate position; h is the height of ith cylinder.

3.4.2 Crown Generation Module

The process of crown generation is shown in Fig. 3. Firstly, the crown length is divided into n (n is an integer) parts on average. The points Ai (i=0, 1, …, n) on the tree stem are defined as start positions of first branch growth and the corresponding points on the crown profile curve are determined as the end positions. The first branches are defined as straight, and the angle between first branches and the stem is α (60°~90°). According to the plant growth principle, first branches are born on the stem and can be defined as Eq. 11.

$$D_2 = \bigcup\left(T_0 \rightarrow T_q\right)\{BD(p, l, r) * \delta(\alpha, \beta)\} \quad (11)$$

Where, D_2 is the first branch layer semantic model; T_0-T_1-T_2-…-T_q are first branches; $BD(p, l, r)$ represents properties of first branches, p is the growth position on the stem, l is the branch length, r is the branch radius; $\delta(\alpha, \beta)$ represents the relationship between first branches and stem, β is the rotational angle around the stem.

The secondary branches are born on the first branches, and are iteratively generated by plant branching rules. The secondary branches are defined as Eq. 12.

$$D_3 = \bigcup_{i=0}^{q-1} \bigcup_{j=0}^{w-1} BD_{ij}(p, l, r) * \delta_{ij}(\alpha, \beta) \quad (12)$$

Where, q is the number of first branches; w is the number of secondary branches; $BD_{ij}(p, l, r)$

represents properties of jth secondary branch from ith first branch; $\delta_{ij}(\alpha, \beta)$ represents the relationship between ith branch and jth branch.

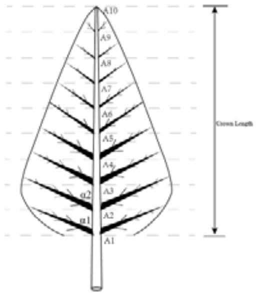

Fig.3 The Process of Crown Generation.

3.4.3 Leaves (Cluster) Generation Module

The leaves are the distinguishing features of different tree species. The leaves semantic model is defined as Eq. 13.

$$D_4 = \sum D_3 * p_0(x, y, z) * \sum_{j=0}^{v} LD(i, l, w, v, p_1) * \delta_j(\beta, \gamma) \quad (13)$$

Where, D_4 is the leaves (cluster) layer semantic model; $p_0(x, y, z)$ is the coordinate position of leaves; $LD(i, l, w, v, p_1)$ are properties of leaves, i is the texture template, l and w represent length and width of leaves cluster polygon, v represents normal vector of the polygon, p_1 is the coordinate position of leaves cluster, $\delta_j(\beta, \gamma)$ represents the relationship between jth leaves cluster and branches.

4. Results and Discussion

4.1 Crown Profile Model Fitting

The fitting of crown profile models was implemented with the Proc Model of SAS statistical software. The parameter q of the weighted function was calculated by crown profile data of three age groups (i.e., Group I, Group II, Group III). When the values of q were 2.1478, 2.3730 and 1.6602 respectively, the heteroscedasticity under different groups was eliminated well. To account for the autocorrelation between adjacent crown measurements, we compared the fitting effects of CAR(1) and CAR(2) Parameter estimates. The associated standard errors and other crown profile models fitting indicators were shown in Table 2.

Table 2 Fitting Results Of Crown Profile Models Based on Car(X) and Nls.

Groups	Models	Model fitting indicators				
		parameters	Estimated values	Standard Errors	*RMSE*	R^2
Group I	NLS	a_0	0.6515	0.0132	0.1395	0.9015
	CAR(1)	a_0	0.5900	0.0336	0.1393	0.9021
		p_1	-0.0412	0.0224		
	CAR(2)	a_0	0.6517	0.0133	0.1402	0.9013
		p_1	0.7281	--		
		p_2	0.0001	--		
Group II	NLS	b_0	0.3640	0.0263	0.2018	0.8768
		b_1	-2.1033	0.0741		
	CAR(1)	b_0	0.0796	0.0407	0.1870	0.8947
		b_1	-1.2200	0.1225		
		p_1	0.1773	0.0204		
	CAR(2)	b_0	0.3640	0.0264	0.2027	0.8760
		b_1	-2.1035	0.0745		
		p_1	0.0001	--		
		p_2	0.0001	--		
Group III	NLS	c_0	-0.2331	0.0301	0.1967	0.8921
		c_1	1.0744	0.1832		
		c_2	-0.8124	0.0628		

	CAR(1)	c_0	-0.2345	0.0308	0.1964	0.8925
		c_1	1.0392	0.1989		
		c_2	-0.8213	0.0663		
		p_1	0.0291	0.0431		
	CAR(2)	c_0	-0.2362	0.0305	0.1977	0.8921
		c_1	1.0833	0.1853		
		c_2	-0.8141	0.0634		
		p_1	0.1131	--		
		p_2	0.0001	--		

Note: p_k is the k-order continuous autoregressive parameter.

For the three different age groups, whereas the R2 was higher and RMSE lower for the CAR(1) than for the other two methods. It can be seen that the CAR(1) improved the prediction effects and performed best.

4.2 Crown Profile Model Validating

We used the cross-validation method to test the prediction accuracy of the crown profile models fit by the CAR(1). The statistical results were shown in Table 3. RMSEs of three models were all less than 0.2m and MAEs were all less than 0.14m. Absolute of MDs were all less than 0.04m. Therefore these results showed that the CAR(1) were effective in predicting the crown radius.

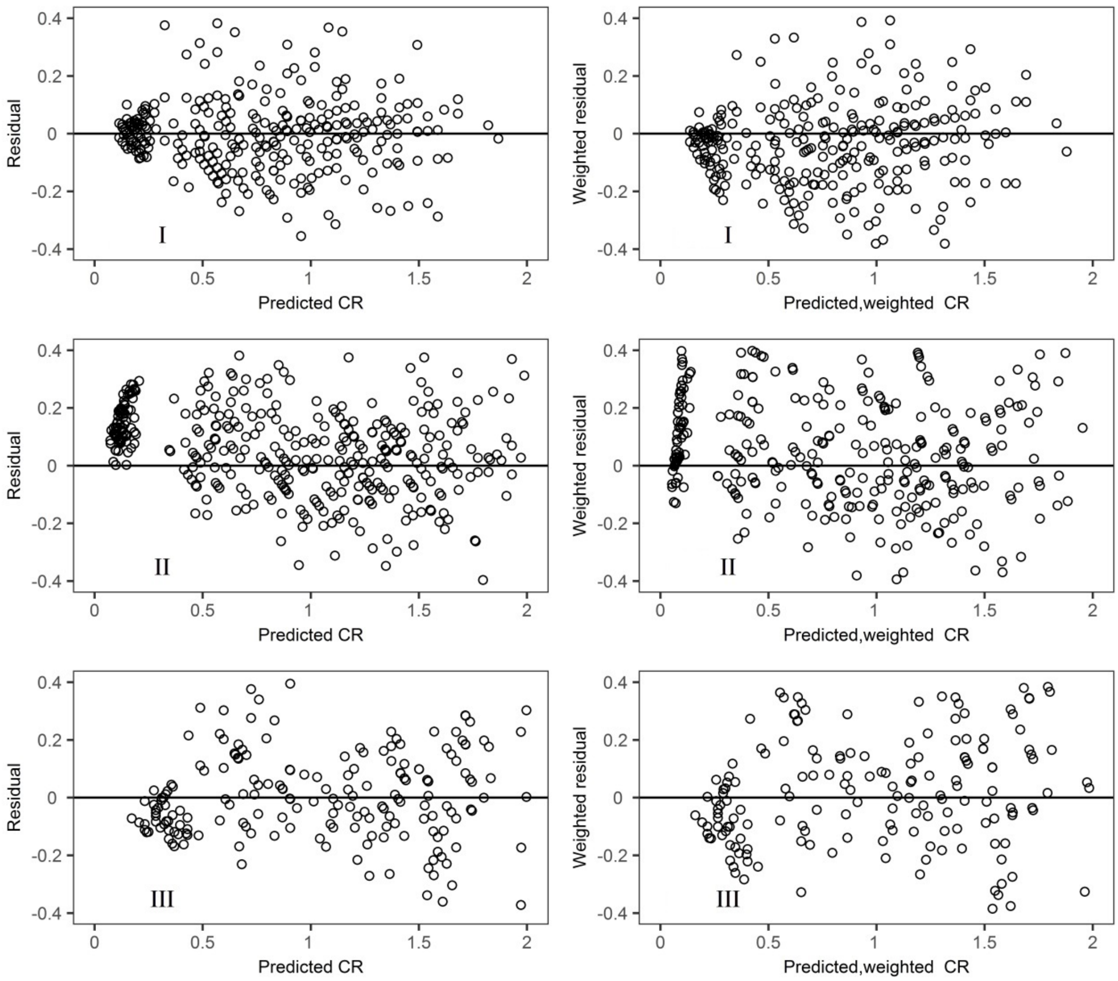

Fig. 4.

Weighted residual plots against predicted values for three crown profile models.

Furthermore, we analyzed heteroscedasticity by the plots of weighted residuals against predicted values for crown profile models (Fig. 4). For Group I, the plots showed that residuals were uniformly distributed above and below 0-level line and well-distributed. This indicated that the heteroscedastictity models used to estimate weights were effective.

Table 3 Test Results Of Crown Profile Models Based on Car(1).

Groups	Models	*MSE*	*RMSE*	*MAE*	*MD*
Group I	CAR(1)	0.0183	0.1353	0.0948	-0.0333
Group II	CAR(1)	0.0347	0.1863	0.1293	0.0088
Group III	CAR(1)	0.0386	0.1965	0.1396	0.0280

4.3 Tree Visualization

Taking the 15 years old Chinese fir in Fujian Province as an example, according to the principle of 3D digital tree model generation, OpenGL graphical interface was used to realize 3D visualization of forest. The Chinese fir generation results were shown in Fig. 5. (a) was the 3D rendering of stem, and (b-c) was the 3D rendering of crown backbone, and (d) was the tree with texture of leaves.

Fig.5 3d Tree Generation Results of Chinese Fir.

In order to vividly display the growth characteristics of crown shapes in forest stand, one sample plot (30m×30m) was selected and 180 trees were simulated. To describe the forest spatial structure, we used the tree positions and measurement factors to render a 3D scene of the forest stand. The rendering of forest stand scene was shown in Fig. 6. All the tree crown shapes and the spatial structure between adjacent trees were displayed. This can express the dynamic growth and change rule of tree crown, and reflect the growth of individual trees and the competition level among different trees.

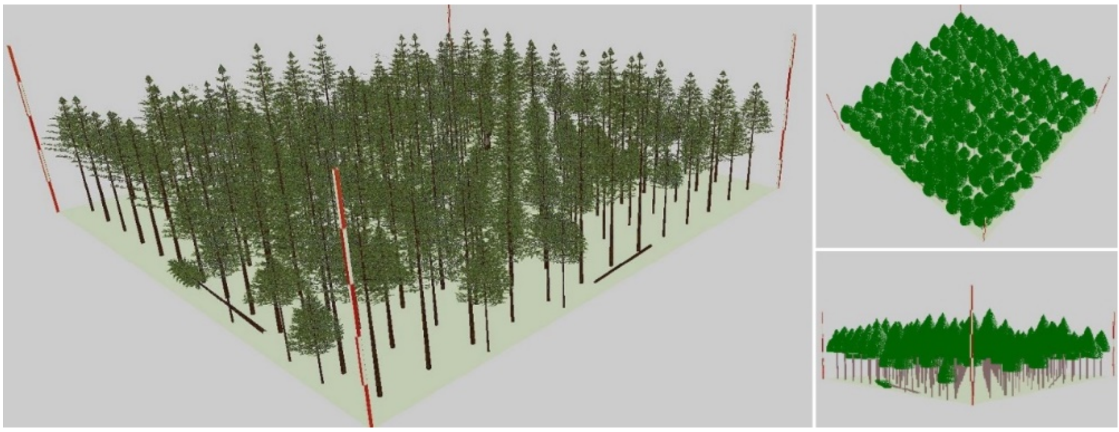

Fig.6 The Forest Stand Visualization of 15-Year- Old Chinese Fir.

5. Conclusion

The crown profile models developed in this study not only performed higher prediction accuracy but also had some variables easily measured. However, the effects of forest management variables on tree crown shapes were not considered. Further studies to relate crown shape to thinning parameters would provide further insights into forest management. The construction of the accurate crown profile models is also of great significance for crown volume calculation, biomass estimation, and forest 3D visualization. Based on the developed crown profile models, we proposed a new approach to construct a digital 3D model to express each tree in forest stand. The 3D digital tree constructed in this paper was more consistent with the actual growth of plantation and was of great significance for qualitative and quantitative analysis of tree competition level.

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